

## INCREASING POWER SYSTEM RELIABILITY USING MINIMUM DYNAMIC LINE RATING

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Anahtar Keywords	Abstract
<i>Robust optimization, mixed integer programming, power system reliability, dynamic line rating, optimal power flow</i>	<i>Due to increase in electricity demand and renewable resources penetration, power transmission infrastructure is utilized close to its technical limits. Dynamic line rating is one of the new practices in power transmission management by which existing transmission lines can be more efficiently utilized. In the literature, dynamic rating is usually applied to all lines of the power system which is not desirable as it is risky to simultaneously monitor a high number of transmission lines. In this paper we propose to apply dynamic rating on few and most suitable lines to effectively increase the system N-K reliability. We develop a robust min-max-min three-layer optimization model to find minimum number of lines for dynamic rating. We use duality theory and convert the three-layer problem into a two-layer min-max problem and then we develop a benders decomposition framework to solve the two-layer problem. We use two test power systems to demonstrate our solution approach and analyze the results.</i>

## MİNİMUM DİNAMİK İLETİM HAT DERECELENDİRMESİ KULLANARAK GÜÇ SİSTEMİN GÜVENİLİRLİĞİNİ ARTIRMA

Kelimeler	Öz
<i>Dayanaklı optimizasyon, karışık tamsayı programlama, güç sistemi güvenilirliği, dinamik iletim hat derecelendirmesi, optimum güç akışı</i>	<i>Hem elektrik talebinde hem de yenilenebilir kaynakların kullanımındaki artış nedeniyle elektrik güç iletim altyapısı teknik sınırlarına yakın bir seviyede kullanılmaktadır. Dinamik iletim hat derecelendirmesi, mevcut iletim hatlarının daha verimli kullanılabileceği güç iletim yönetimindeki yeni uygulamalardan biridir. Literatürde dinamik derecelendirme genellikle güç sisteminin tüm hatlarına uygulanır ve bu yaklaşım, güç sistemi yöneticisini çok sayıda iletim hattını aynı anda izlemeye zorlar ve sistemin güvenilirliğini tehlikeye sokar. Bu çalışmada, sistemin N-K güvenilirliğini etkili bir şekilde artırmak için çok az sayıda ve en uygun iletim hatlarına dinamik derecelendirme uygulanması önerilmektedir. Dinamik derecelendirme için minimum sayıda iletim hatları bulmak için gürbüz bir min-max-min üç katmanlı optimizasyon modeli geliştirilmiştir. İkili teorisini kullanarak üç katmanlı problem iki katmanlı min-max probleme dönüştürülmüştür ve ardından iki katmanlı problemi çözmek için bir benders ayrıştırma (decomposition) yöntemi geliştirilmiştir. Çözüm yaklaşımı uygulamak ve sonuçları analiz etmek için iki test güç sistemi kullanılmıştır.</i>

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## 1. Introduction

Determining which part of power system to invest to improve the reliability is nowadays more important as power systems are experiencing a huge transition from fossil fuel-based power plants to renewable energy-based power generators. This transition leads to power systems with mixed sources which can be detrimental to the reliability of power system subject to static power transmission network. Dynamic Line Rating (DLR), on the other hand, can increase the efficiency and the reliability of power systems. The dynamic line rating is one of the new practices in power transmission network and calculates the thermal capacity of lines based on the real-time line condition and surrounding ambient factors (Ciniglio and Deb 2004, TWENTIES (Transmission system operation with large penetration of Wind and other renewable Electricity sources in Networks by means of innovative Tools and Integrated Energy Solutions) 2013). Technologies developed for dynamic line rating and real-time monitoring are reviewed in the literature (Fernandez, E., Albizu, I., Bedialauneta, M.T., Mazon, A.J., Leite, P.T. 2016), where advantages and disadvantages of applying dynamic rating on transmission systems are studied. An approach is proposed (Fernandes, S. W., Rosa, M. A., Issicaba, D. 2020) to increase robustness and safety of dynamic line rating system using state estimation and bad data analysis algorithms.

Different forecasting models are developed in the literature to predict the uncertain dynamic ratings. A forecasting model is developed (Abboud et al. 2019) to predict dynamic line ratings where the high-resolution rapid refresh model is integrated with computational fluid dynamics. Also, probabilistic day-ahead forecasting methods for dynamic line rating is developed (Dupin, R., Kariniotakis, G., Michiorri, A. 2019). Different machine-learning methods (random forests and quantile random forests, multivariate adaptive regression splines, generalized linear models) are deployed to predict dynamic line ratings (Aznarte and Siebert 2017). These studies indicate considerable enhancement for point and probabilistic forecasts. In another study (S. Madadi, B. Mohammadi-Ivatloo and S. Tohidi 2020), the Ornstein-Uhlenbeck process is extended into an integrated factorized method to model and predict dynamic line rating values by considering hidden factors of dynamic rating such as the weather conditions. Quantile regression and

superquantile regression methods are also used to predict dynamic line rating in operational applications with very short-term horizons (Kirilenko, Esmaili and Chung 2021).

Dynamic line rating is considered in the unit commitment problem (Nick, Alizadeh-Mousavi, Cher-kaoui and Paolone 2016, Hemparuva, Simon, Kinat-tingal, Padhy 2018, Li et al. 2018, Park, H., Jin, Y. G., Park, J. K. 2018, Uman, Feng, Abbas, Habib, Hao 2021) to characterize its contribution and effectiveness on the planning of operations in the power systems with renewable resources. Considering dynamic line rating in the unit commitment problem decreases the cost of electricity generation operations.

Dynamic line rating is also proposed to be applied by the power systems to increase the penetration of the wind power. For instance, a new model is proposed to investigate the effectiveness of the dynamic line rating to decrease the amount of network congestion and also decrease the wind power curtailment in power systems (Banerjee, B., Jayaweera, D., Islam, S. 2015). Further, the use of dynamic line rating systems in wind integration is reviewed in the literature (Fernandez et al. 2016). In another study (Teng et al. 2018), the power systems with renewable resources (wind) are considered and the effectiveness of dynamic line rating is analyzed. In their model a probabilistic forecasting approach is used for the dynamic rating and a two-stage stochastic optimization model is developed.

Considering the real time ambient conditions results in better utilization of the transmission elements during the contingency times and improves the overall reliability. However, there is limited work in the literature that utilizes DLR (Dynamic Line Rating) to improve the reliability. Dynamic line rating along with optimal transmission switching are integrated into the security assessment of the electricity production (Xiao, R., Xiang, Y., Wang, L., and Xie, K. 2016) to investigate and study their influence on the reliability of bulk power grids. Their result show that utilizing these technologies improve the reliability. In another work (Zhang, S., Liu, C., Gu, X., and Wang, T. 2017) dynamic line rating is incorporated into a power system dispatch problem with transmission switching to intensify system security. Their goal is to adjust the distribution of power flow and make it more homogeneous while satisfying the single contingency (N-1) scenarios and to alleviate

transmission congestion. Another methodology is developed (Banerjee, B., Jayaweera, D., and Islam, S. M. 2014) to measure the un-used scheduling capacity of a power system subject to uncertain dynamic rating and intermittent wind power. In power systems with high amounts of available wind energy, there is always the potential for the network congestion and their goal is to mitigate this risk. Dynamic line rating is also incorporated in a chance constraint program (Bucher, M. A., Vrakopoulou, M., and Andersson, G. 2013) where the single contingency (N-1) security scenarios with a probabilistic assessment approach is proposed. Their studies show that capacity of the transmission network can be more utilized, and, at the same time, the single contingency reliability can be reached.

In the cited studies so far, dynamic line rating is applied to all lines in the system which is not desirable as it is risky to simultaneously monitor a high number of transmission lines. There are few works in the literature (Qiu and Wang 2015, Garifi and Baker 2020) that apply dynamic line rating on few and critical lines. A congestion management model utilizing chance constraints is proposed (Qiu and Wang 2015) where dynamic ratings are applied on critical lines. In their model a prespecified value is used as the upper bound for the chance of the line overload. Another integer chance constrained power dispatch problem with dynamic line ratings is proposed (Garifi and Baker 2020) to manage line overloads and minimize curtailment of wind energy. Their experiments on a test power system reduces the total average wind power curtailment by 15%. In these studies, the risk of line overload is considered but the risk of contingencies such as N-K reliability concerns are not considered while selecting the critical lines for dynamic rating.

In this paper we propose to apply dynamic rating on few and most suitable lines to effectively increase the system security against N-K reliability concerns. Usually considering N-K reliability requirements is computationally expensive. We propose a min-max-min robust optimization framework to secure the power system against the worst-case N-K reliability scenario as much as possible. This way of modeling requires much smaller number of constraints and decision variables and, therefore, is computationally more tractable. We develop a benders decomposition approach to solve the proposed min-max-min model. The master problem is the outer minimization problem where critical lines are

selected for the economic power dispatch. The slave problem is the inner max-min problem and evaluates the N-K reliability of the system subject to selected lines for dynamic rating.

Our paper is summarized as following: In Section 2, we propose our min-max-min model. In Section 3, we describe our solution approach and the way we solve the proposed min-max-min problem. In Section 4 we demonstrate our model and solution approach on two test power systems. Finally, Section 5 concludes the paper.

## 2. Proposed Model

This study complies with the research and publication ethics (Bu çalışmada araştırma ve yayın etiğine uyulmuştur). Our model in this paper is based on well-known optimal power flow problem in the power systems. In power systems we have three core elements: generators, transmission lines, and demand sites (or loads). Generators produce electric power that we need. Transmission lines transmit the electric power produced by the generators to demand sites. The demand sites (or loads) consume the produced electric power. The place where generators and transmission lines and loads are connected is called a bus. A power system can have many generators, lines, loads, and buses. In this paper, we use  $g_n$  to denote the amount of power produced by generator  $n$ . The amount of power produced by each generator cannot be less than a minimum level and, also, cannot be more than a maximum level. We use  $G_n^{\min}$  and  $G_n^{\max}$  to denote these boundaries. We use  $f_m$  to denote the amount of power transmitted by line  $m$ . Please note that the amount of power transmitted by a line cannot exceed a certain maximum transmission amount because of the security reasons. This maximum transmission amount is called line rating. Traditionally, static rating is used to determine the maximum transmission for a line which is very conservative. Recently, dynamic rating is proposed to determine the maximum transmission more efficiently. We use  $F_m^s$  to denote static rating of line  $m$ . We also use  $F_m^d$  to denote dynamic rating of line  $m$ . We use  $D_b$  to denote the amount of power demand at bus  $b$ . In power systems, phase angle is the angle of the voltage with reference to time. We use  $\theta_b$  to denote voltage angle at bus  $b$ . The voltage angle difference between two buses should not exceed a certain amount. We use  $\Theta_m^{\max}$  to denote the maximum phase angle difference between origin and destination buses of line  $m$ . In

power systems, if the amount of produced power exceeds the demand, then the the extra produced power is curtailed and is not used. We use  $s_b^-$  to denote the amount of generated power curtailed at bus  $b$ . On the other hand, if the amount of produced power is less that the demand, then the extra demand for power is curtailed and is not satisfied. We use  $s_b^+$  to denote the amount of power demand curtailed at bus  $b$ . The summary of notations used in our model are listed below:

$nl$  Number of lines.

$ng$  Number of generators.

$nb$  Number of buses.

Sets:

$L = \{1, \dots, nl\}$  Set of lines.

$\psi_b^- \subseteq L$  Set of lines consuming power from bus  $b$ .

$\psi_b^+ \subseteq L$  Set of lines injecting power to bus  $b$ .

$G = \{1, \dots, ng\}$  Set of generators.

$\eta_b \subseteq G$  Set of generators at bus  $b$ .

$B = \{1, \dots, nb\}$  Set of buses.

Indices:

$m \in L$  Lines.

$n \in G$  Generators.

$b \in B$  Buses.

$a_m$  Origin bus for line  $m$ .

$b_m$  Destination bus for line  $m$ .

$\gamma_n$  Bus index of generator  $n$ .

Parameters:

$c_b^-, c_b^+$  Generation and load curtailment costs at bus  $b$ , respectively.

$c_n^g$  Operational cost of generator  $n$ .

$c_m^x$  Investment cost in dynamic rating for line  $m$ .

$D_b$  Electricity demand at bus  $b$ .

$Y_m$  Electrical susceptance of line  $m$ .

$G_n^{\min}, G_n^{\max}$  Min and max capacity of generator  $n$ , respectively.

$\Theta_m^{\max}$  Max phase angle difference between origin and destination buses of line  $m$ .

$F_m^s$  Static rating of line  $m$ .

$F_m^d$  Dynamic rating of line  $m$ .

$M$  A sufficiently large number.

$K$  Number of contingencies in generators or lines.

$J$  maximum number of lines with dynamic rating

Decision Variables:

$g_n, g_n^0$  Power generated by generator  $n$ .

$f_m, f_m^0$  Real power flow transmitted by line  $m$ .

$\theta_{a_m}, \theta_{a_m}^0$  Voltage angle at origin bus for line  $m$ .

$\theta_{b_m}, \theta_{b_m}^0$  Voltage angle at destination bus for line  $m$ .

$s_b^-, s_b^+$  Generation and load curtailments at bus  $b$ , respectively.

$x_m$  Binary decision variable representing dynamic rating status of line  $m$  (0 not utilized, 1 utilized).

$r_m, r_n'$  Binary decision variables representing contingency states of line  $m$  and generator  $n$ , respectively (0 out of service, 1 in service).

In this section we introduce our three-layer min-max-min model. First, we describe the inner min problem (1), then we explain the max-min problem (2), finally we describe the min-max-min problem (3).

The goal of inner min problem (1) is to minimize total generation and load curtailment throughout the system. The inner min problem (1) is an extension of the well-known DCOPF (Direct Current Optimal Power Flow) model in the literature where the dynamic line rating is allowed to be applied in certain preselected lines. The inner min problem (1) is described by equations (1a) to (1j). The objective function (1a) minimizes total generation and load curtailments. Constraints (1b) ensure that the power flowing into each bus equals the power flowing out of each bus. The physical relation between voltage angles of connected buses and the power flow in connecting lines is represented by constraints (1c) and (1d). Thermal limits of lines are respected by constraints (1e) and (1f) and generation capacities of generators are assured by constraints (1g). The bounds on the phase angle differences of connected buses are enforced by constraints (1h). The  $r_m$  and  $r_n'$  in constraints (1e), (1f) and (1g) are constants and their values are already determined by the max-min problem (2).

The inner min problem (1)

$$\text{Min } P_1 = \sum_{b \in B} (c_b^- s_b^- + c_b^+ s_b^+) \quad (1a)$$

Subject to

$$\sum_{n \in \eta_b} g_n + \sum_{m \in \psi_b^+} f_m - \sum_{m \in \psi_b^-} f_m - s_b^- + s_b^+ = D_b \quad \forall b \in B \quad [\alpha_b] \quad (1b)$$

$$f_m - Y_m(\theta_{a_m} - \theta_{b_m}) \geq -(1 - r_m)M \quad \forall m \in L \quad [\phi_m^-] \quad (1c)$$

$$f_m - Y_m(\theta_{a_m} - \theta_{b_m}) \leq (1 - r_m)M \quad \forall m \in L \quad [\phi_m^+] \quad (1d)$$

$$f_m \geq -(F_m^s(1 - x_m) + F_m^d x_m)r_m \quad \forall m \in L \quad [\varphi_m^-] \quad (1e)$$

$$f_m \leq (F_m^s(1 - x_m) + F_m^d x_m)r_m \quad \forall m \in L \quad [\varphi_m^+] \quad (1f)$$

$$G_n^{\min} r_n' \leq g_n \leq G_n^{\max} r_n' \quad \forall n \in G \quad [\omega_n^-, \omega_n^+] \quad (1g)$$

$$-\Theta_m^{\max} \leq \theta_{a_m} - \theta_{b_m} \leq \Theta_m^{\max} \quad \forall m \in L \quad [\delta_m^-, \delta_m^+] \quad (1h)$$

$$g_n, s_b^-, s_b^+ \geq 0 \quad \forall (b, n) \quad b \in B, \quad n \in G \quad (1i)$$

$$f_m, \theta_b \quad \text{unrestricted} \quad \forall (b, m) \quad b \in B, \quad m \in L \quad (1j)$$

The max-min problem (2)

$$\text{Max Min } P_1 \tag{2a}$$

Subject to

$$\sum_{m \in L} (1 - r_m) + \sum_{n \in G} (1 - r'_n) = K \tag{2b}$$

$$r_m, r'_n \in \{0,1\} \quad \forall(m,n) \quad m \in L, \quad n \in G \tag{2c}$$

The max-min problem (2) is described by equations (2a) to (2c). In other words, the  $r_m$  and  $r'_n$  are constants in the inner min problem (1) but they are decision variables in the max-min problem (2). First, the max-min problem (2) is solved and the optimal

values for the  $r_m$  and  $r'_n$  are determined. Then the obtained optimal values for the  $r_m$  and  $r'_n$  are replaced in the inner min problem (1) as constants and the inner min problem (1) is then solved. In the max-min problem (2) the binary decision variables  $r_m$  and  $r'_n$  are used to maintain the system reliability at K contingencies in generators or transmission lines (the N-K reliability requirements). A value of  $r_m = 0$  means that transmission line  $m$  is under contingency and therefore it is not working. Similarly, a value of  $r'_n = 0$  means generator  $n$  is not working. If  $K = 1$  then single contingencies (N-1 reliability requirements) are considered. Note that one can consider contingencies with simultaneous failure of multiple elements (e.g. N-2 or others) by setting  $K > 1$ .

The min-max-min problem (3)

$$\text{Min } \sum_{n \in G} c_n^g g_n^0 + \sum_{m \in L} c_m^x x_m + \text{Max Min } P_1 \tag{3a}$$

Subject to

$$\sum_{n \in \eta_b} g_n^0 + \sum_{m \in \psi_b^+} f_m^0 - \sum_{m \in \psi_b^-} f_m^0 = D_b \quad \forall b \in B \tag{3b}$$

$$f_m^0 = Y_m(\theta_{a_m}^0 - \theta_{b_m}^0) \quad \forall m \in L \tag{3c}$$

$$f_m^0 \geq -F_m^s(1 - x_m) - F_m^d x_m \quad \forall m \in L \tag{3d}$$

$$f_m^0 \leq F_m^s(1 - x_m) + F_m^d x_m \quad \forall m \in L \tag{3e}$$

$$G_n^{\min} \leq g_n^0 \leq G_n^{\max} \quad \forall n \in G \tag{3f}$$

$$-\Theta_m^{\max} \leq \theta_{a_m}^0 - \theta_{b_m}^0 \leq \Theta_m^{\max} \quad \forall m \in L \tag{3g}$$

$$\sum_{m \in L} x_m \leq J \tag{3h}$$

$$g_n^0 \geq 0 \quad \forall n \in G \tag{3i}$$

$$f_m^0, \theta_b^0 \text{ unrestricted} \quad \forall(b,m) \quad b \in B, \quad m \in L \tag{3j}$$

$$x_m \in \{0,1\} \quad \forall m \in L \tag{3k}$$

The optimal solution for the max-min problem (2) is the worst-case contingency scenario for the power system in terms of curtailed generation and load. The left and right hand sides of constraints (1e) and (1f) are multiplied by binary variable  $r_m$  to ensure there is no power flow in lines that are out of service due to the contingency. Similarly, the left and right hand sides of constraints (1g) are multiplied by binary variable  $r'_n$  to ensure there is no power generation in generators that are out of service due to the contingency. If a line is in service ( $r_m = 1$ ), then inequalities (1c) and (1d) will be equivalent to the equality  $f_m = Y_m(\theta_{a_m} - \theta_{b_m})$ . On the other hand, if a line is not in service ( $r_m = 0$ ), then inequalities (1c) and (1d) will be independent of power flow variable  $f_m$  and will be redundant constraints.

Finally, the three-layer min-max-min problem (3) is described by equations (3a) to (3k). The  $c_n^g$  is operational cost of generator  $n$  and  $c_m^x$  is the cost of dynamic rating for line  $m$ . The main decision variable in the three-layer min-max-min problem (3) is  $x_m$  which is the dynamic rating status of line  $m$ . The  $x_m = 1$  means dynamic rating is applied to line  $m$  and  $x_m = 0$  means static rating is applied to line  $m$ . All constraints are similar to the constraints in problem (1) except constraint (3h). Constraint (3h) limits the number of lines with dynamic rating to

maximum  $J$  lines which is the main drive of this paper: to limit dynamic rating to few and most suitable transmission lines. Therefore, problem (3) explores and finds the minimum number of candidate lines for dynamic rating such that total generation and dynamic line rating cost is reduced, and, in addition, the aftermath of the worst-case N-K contingency scenario is minimized. In other words, the solution of min-max-min problem (3) is economic and is robust against all N-K contingencies and guarantees the reliability of the power system.

### 3. Proposed Solution Approach

The min-max-min problem (3) described in previous section has three layers and directly cannot be solved by commercial solvers. In this section we propose a solution approach to solve this problem. In first step of our proposed solution approach, we take the dual of the inner min problem (1). As the dual of a minimization problem is a maximization problem, therefore, the inner min problem becomes a dual max problem. This dualization also converts the primal max-min problem (2) into a dual max-max problem or simply a dual max problem. The resulting dual max problem is described by equations (4a) to (4j).

#### The dual max problem (4)

$$\begin{aligned} \text{Max } DP_1 = & \sum_{b \in B} D_b \alpha_b + \sum_{m \in L} (-(1 - r_m)M\phi_m^- + (1 - r_m)M\phi_m^+) \\ & + \sum_{m \in L} -(F_m^s(1 - x_m) + F_m^d x_m)r_m \varphi_m^- + \sum_{m \in L} (F_m^s(1 - x_m) + F_m^d x_m)r_m \varphi_m^+ \\ & + \sum_{n \in G} (G_n^{\min} r'_n \omega_n^- + G_n^{\max} r'_n \omega_n^+) + \sum_{m \in L} (-\Theta_m^{\max} \delta_m^- + \Theta_m^{\max} \delta_m^+) \end{aligned} \tag{4a}$$

Subject to

$$\alpha_{\gamma_n} + \omega_n^- + \omega_n^+ \leq 0 \quad \forall n \in G \tag{4b}$$

$$-\alpha_{a_m} + \alpha_{b_m} + \phi_m^- + \phi_m^+ + \varphi_m^- + \varphi_m^+ = 0 \quad \forall m \in L \tag{4c}$$

$$\sum_{m \in \psi_b^+} (-Y_m \phi_m^- - Y_m \phi_m^+ + \delta_m^- + \delta_m^+)$$

$$+ \sum_{m \in \psi_b^-} (Y_m \phi_m^- + Y_m \phi_m^+ - \delta_m^- - \delta_m^+) = 0 \quad \forall b \in B \tag{4d}$$

$$-c_b^- \leq \alpha_b \leq c_b^+ \quad \forall b \in B \tag{4e}$$

$$\sum_{m \in L} (1 - r_m) + \sum_{n \in G} (1 - r'_n) = K \tag{4f}$$

$$\phi_m^-, \varphi_m^-, \omega_n^-, \delta_m^- \geq 0 \quad \forall(m, n) \quad m \in L, \quad n \in G \tag{4g}$$

$$\phi_m^+, \varphi_m^+, \omega_n^+, \delta_m^+ \leq 0 \quad \forall(m, n) \quad m \in L, \quad n \in G \tag{4h}$$

$$\alpha_b \text{ unrestricted} \quad \forall b \in B \tag{4i}$$

$$r_m, r'_n \in \{0,1\} \quad \forall(m, n) \quad m \in L, \quad n \in G \tag{4j}$$

In problem (4) the set of dual variables are  $\Omega = \{\alpha, \phi^-, \phi^+, \varphi^-, \varphi^+, \omega^-, \omega^+, \delta^-, \delta^+\}$ . The solution space of problem (4) is linear which is described by constraints (4b)– (4j). However, there are bilinear terms  $r_m \phi_m^-, r_m \phi_m^+, r_m \varphi_m^-, r_m \varphi_m^+, r'_n \omega_n^-$  and  $r'_n \omega_n^+$  in the objective function of this problem. These terms make problem (4) a mixed integer bilinear problem.

In this paper we use a linearization technique and linearize the nonlinear problem (4) to a mixed integer linear program. In our linearization technique, we replace bilinear term  $r_m \phi_m^-$  with auxiliary variable  $u_m^-$  and add constraints (5a) and (5b) to problem (4).

$$-M(1 - r_m) \leq u_m^- - \phi_m^- \leq M(1 - r_m) \quad \forall m \in L \tag{5a}$$

$$0 \leq u_m^- \leq M r_m \quad \forall m \in L \tag{5b}$$

Similarly, we replace the remaining bilinear terms  $r_m \phi_m^+, r_m \varphi_m^+, r'_n \omega_n^-, r'_n \omega_n^+$  with auxiliary variables  $u_m^+, v_m^-, z_n^-, z_n^+$ , respectively, and linearize them as well. We define  $\Omega' = \{u_m^-, u_m^+, v_m^-,$

$v_m^+, z_n^-, z_n^+\}$  as the set of new auxiliary variables. The updated problem (4) is described by equations (6a) to (6n).



The linearized dual max problem (6)

$$\begin{aligned} \text{Max } LDP_1 = & \sum_{b \in B} D_b \alpha_b + \sum_{m \in L} M_m (-\phi_m^- + u_m^- + \phi_m^+ - u_m^+) \\ & + \sum_{m \in L} -(F_m^s(1 - x_m) + F_m^d x_m) v_m^- + \sum_{m \in L} (F_m^s(1 - x_m) + F_m^d x_m) v_m^+ \\ & + \sum_{n \in G} (G_n^{\min} z_n^- + G_n^{\max} z_n^+) + \sum_{m \in L} (-\Theta_m^{\max} \delta_m^- + \Theta_m^{\max} \delta_m^+) \end{aligned} \tag{6a}$$

Subject to

$$\text{Constraints (4b)-(4j)} \tag{6b}$$

$$-M(1 - r_m) \leq u_m^- - \phi_m^- \leq M(1 - r_m) \quad \forall m \in L \tag{6c}$$

$$0 \leq u_m^- \leq Mr_m \quad \forall m \in L \tag{6d}$$

$$-M(1 - r_m) \leq u_m^+ - \phi_m^+ \leq M(1 - r_m) \quad \forall m \in L \tag{6e}$$

$$-Mr_m \leq u_m^+ \leq 0 \quad \forall m \in L \tag{6f}$$

$$-M(1 - r_m) \leq v_m^- - \phi_m^- \leq M(1 - r_m) \quad \forall m \in L \tag{6g}$$

$$0 \leq v_m^- \leq Mr_m \quad \forall m \in L \tag{6h}$$

$$-M(1 - r_m) \leq v_m^+ - \phi_m^+ \leq M(1 - r_m) \quad \forall m \in L \tag{6i}$$

$$-Mr_m \leq v_m^+ \leq 0 \quad \forall m \in L \tag{6j}$$

$$-M(1 - r'_n) \leq z_n^- - \omega_n^- \leq M(1 - r'_n) \quad \forall n \in G \tag{6k}$$

$$0 \leq z_n^- \leq Mr'_n \quad \forall n \in G \tag{6l}$$

$$-M(1 - r'_n) \leq z_n^+ - \omega_n^+ \leq M(1 - r'_n) \quad \forall n \in G \tag{6m}$$

$$-Mr'_n \leq z_n^+ \leq 0 \quad \forall n \in G \tag{6n}$$

The linearized dual max problem (6) is equivalent to the max-min problem (2). However, problem (6) is a mixed integer linear program and can be directly solved using any commercial solver. Therefore, instead of the max-min problem (2) we will use

problem (6). Now we develop a benders decomposition approach to solve the min-max-min problem (3). We decompose the problem (3) into a master problem and a slave problem. The master problem is formulated by (7a) to (7b).

The master problem (7)

$$\text{Min } MP = \sum_{n \in G} c_n^g g_n^0 + \sum_{m \in L} c_m^x x_m \quad (7a)$$

Subject to

$$\text{Constraints (3b)–(3k)} \quad (7b)$$

The goal of the master problem (7) is to find the best lines for the dynamic rating practice. The solution of the master problem (7) provides a lower bound for the optimal solution. The variable  $DV$  represents the

objective function of the linearized dual max problem (6). At the beginning of the decomposition algorithm  $DV = 0$ . When the algorithm progresses, the optimality cuts (defined in the following) are gradually added to the master problem (7) which restrict  $DV$  to positive values. When the master problem (7) is solved, its proposed  $x$  solutions are passed to the slave problem. The slave problem is the linearized dual max problem (6). When the slave problem (6) is solved and optimal solutions for  $r, r', \Omega, \Omega'$  are obtained, the benders optimality cut (8) is calculated using the optimal solutions of the slave problem (6) and is added to the master problem (7).

The benders optimality cut (8)

$$\begin{aligned} DV \geq & \sum_{b \in B} D_b \alpha_b + \sum_{m \in L} M_m (-\phi_m^- + u_m^- + \phi_m^+ - u_m^+) \\ & + \sum_{m \in L} -(F_m^s(1-x_m) + F_m^d x_m) v_m^- + \sum_{m \in L} (F_m^s(1-x_m) + F_m^d x_m) v_m^+ \\ & + \sum_{n \in G} (G_n^{\min} z_n^- + G_n^{\max} z_n^+) + \sum_{m \in L} (-\Theta_m^{\max} \delta_m^- + \Theta_m^{\max} \delta_m^+) \end{aligned} \quad (8)$$

Our solution approach is summarized as the following procedure.

1. Set  $LB = -\infty$  and  $UB = +\infty$  where  $LB$  is the lower bound of the decomposition algorithm and  $UB$  is the upper bound.
2. Solve the master problem (7) and calculate the dynamic rating plan  $x$ . Update the lower bound of the problem to the objective value of the master problem (7), i.e.  $LB = MP$ . Pass the proposed plan  $x$  to the slave problem (6).
3. Solve the slave problem (6) and update the upper bound of the problem to sum of the objective values of the master and slave problems, i.e.  $UB = MP + LDP_1$ .
4. If the difference ratio between the upper bound and the lower bound is larger than the pre-specified optimality gap threshold  $\varepsilon$  (i.e. if  $\frac{UB-LB}{UB} \geq \varepsilon$ ), then add the optimality cut (8) to the master problem (7) and return to the step 2. Otherwise, consider the current proposed plan  $x$  as the optimal solution and terminate the decomposition algorithm.

**4. Numerical Experiments**

In this section, we demonstrate our proposed model and solution approach on two test power systems: IEEE (Institute of Electrical and Electronics Engineers) RTS-79 and IEEE (Institute of Electrical and Electronics Engineers) 118-bus system. In all experiments a value of  $\pm 1.2$  radian is considered for minimum/maximum phase angle differences. The lines that are selected for dynamic rating are assigned 50% more transmission capacity. The dynamic rating cost is set to 0.1% of optimal dispatch cost to ensure that uneconomic lines are not selected for dynamic rating.

Generation and load curtailment costs are set to the largest generation cost in the system to give the system reliability more priority than the generation economy. An optimality gap of 0.1% is used to terminate the proposed benders decomposition algorithm.

The C++ is used to develop the benders decomposition algorithm. The Gurobi version 9.0.3

on a computer model with Intel(R) Core(TM) i5-7200U CPU @ 2.50 GHz 2.70 GHz and 8.00 GB of RAM memory is used to solve the models and calculate the results.

**4.1 Experiments on The RTS-79 Power System**

The RTS-79 has 24 buses, 38 lines and 32 generation units. As the data for this system is proposed four decades ago (Subcommittee 1979), its load and generation data should be updated. In this paper we double the load and generation for the RTS-79 to represent today's market size. We also set maximum number of DLR lines to 10 lines.

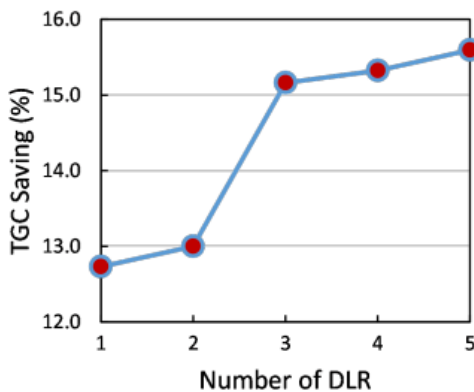
The results are summarized in Table 1. As shown in this table, the identities of lines selected for dynamic rating are almost same for all different values of K. Lines 10, 11, 17, 23 and 28 are the most suitable lines

for dynamic rating. For N-1 and N-5 cases line 29 is also selected. In Table 1, the TGLCC stands for total generation and load curtailment cost in the worst-case scenario. As it can be seen from this table, dynamic line rating considerably decreases TGLCC of the system given different worst-case contingency cases. For instance, in the case of N-1 reliability level, the potential annual saving in TGLCC is equal to  $3564 \times 12.9\% \times 24 \times 365 = \$4,027,463$  which is significant.

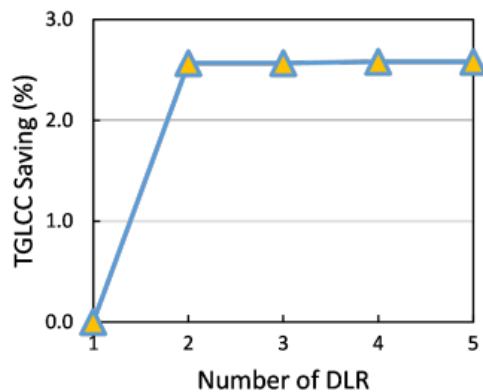
The utilization of dynamic rating also reduces generation cost from \$3700 per hour to \$3123 per hour which is 15.6% improvement. These results are obtained in very short computation time as reported in the last column of Table 1 in seconds. We further investigate the contribution of selected five lines 10, 11, 17, 23 and 28 on TGLCC savings and total generation cost (TGC) savings in N-2 reliability case. The results are summarized in Figures 1a and 1b.

Table 1  
Minimum dynamic rating on different N-K reliability levels in RTS-79 system

K	DLR Lines	TGLCC (\$)	TGLCC Saving (%)	Time (s)
1	10, 11, 17, 23, 28, 29	3564	12.9	2.3
2	10, 11, 17, 23, 28	6906	2.6	8.5
3	10, 11, 17, 23, 28	9620	3.1	25.2
4	10, 11, 17, 23, 28	11300	3.4	82.6
5	10, 11, 17, 23, 28, 29	13278	2.5	402.7



(a)



(b)

Fig. 1. TGC and TGLCC Savings in N-2 reliability case in RTS-79 system

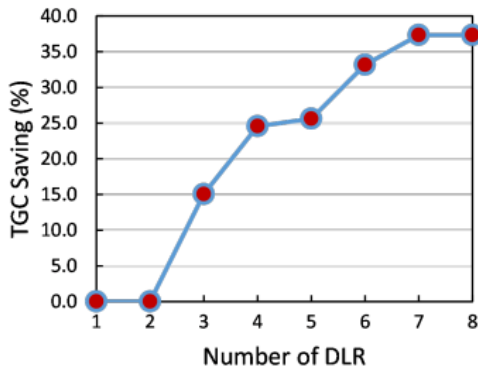
As shown in Figure 1a, applying dynamic rating on one line (line 10) provides most of TGC savings (12.7%) and applying dynamic rating on more than one line does not significantly increase TGC savings. Moreover, we see in Figure 1b that applying dynamic rating on two lines (lines 10 and 11) provides enough TGLCC savings and applying more lines is not necessary. Therefore, we conclude that for the modified RTS-79 system applying dynamic line rating on lines 10 and 11 is satisfactory. This result concludes the main drive of this paper that minimum dynamic line rating is much better than applying dynamic line rating on all lines.

### 4.2 Experiments on The IEEE 118-Bus Power System

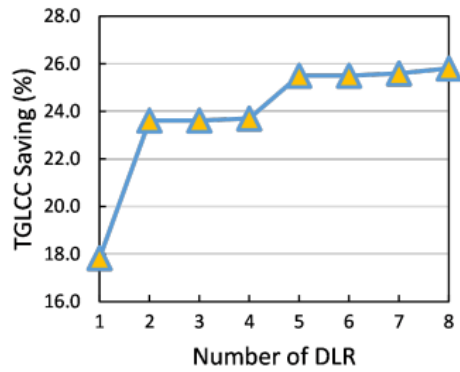
We continue our experiments by applying our proposed model on larger IEEE 118-bus power system. The IEEE 118-bus system has 19 generators and 186 transmission lines. Data for the IEEE 118-bus power system is downloaded from an online source (PSA 1999).

Table 2  
Minimum dynamic rating on different N-K reliability levels in 118-bus system

K	DLR Lines	TGLCC (\$)	TGLCC Saving (%)	Time (s)
1	116, 134, 141, 142, 154	1840	9.4	12.3
2	51, 112, 113, 116, 134, 141, 142, 154	5169	25.8	17.1
3	51, 112, 113, 116, 134, 141, 142, 154	8651	17.2	18.4
4	51, 113, 116, 134, 141, 142, 154	10497	11.3	77.7
5	42, 113, 116, 134, 141, 142, 154	12390	41.3	107.6



(a)



(b)

Fig. 2. TGC and TGLCC Savings in N-2 reliability case in 118-bus system

For this power system, generators' capacity, generation costs, transmission network and line characteristics are taken from a relevant paper (Fisher, E. B., O'Neill, R. P., and Ferris, M. C. 2008). We also add four wind farms to buses 22, 45, 76 and 105 where capacity of each wind farm is 50 MW. We

again set maximum number of DLR lines to 10 lines. The results are summarized in Table 2.

As shown in this table, amounts of savings in generation and load curtailments are significant where the highest saving (41.3%) is observed in N-5 reliability level. Lines 116, 134, 141, 142 and 154 are selected for dynamic rating in all reliability levels.

Maximum number of selected lines for dynamic rating is eight lines which is observed in N-2 and N-3 reliability levels. Therefore, and again, a small fraction of transmission lines (8 out of 186 lines) is selected for dynamic rating and the rest of lines are not needed for dynamic line rating operation. The longest computation time (which occurs in N-5 case) is less than two minutes which is quite reasonable.

Figure 2a illustrates savings in total generation cost for the 118-bus system with N-2 reliability level when different number of lines are selected for dynamic rating. To have savings in total generation cost, at least three lines should be selected for dynamic rating. As number of selected lines increases, the TGC saving increases as well. However, this is not the case in terms of savings in curtailments as shown in Figure 2b. If only one line (line 113) is selected for dynamic rating, then almost 18% of total curtailment cost is saved. If only two lines (lines 51 and 113) are selected for dynamic rating, then almost 24% of total curtailment cost is saved. Therefore, most of saving in generation and load curtailment is achieved when one or two lines are targeted for dynamic rating. The decision maker can refer to Figures 2a and 2b and make trade-offs between number of DLR lines and savings in TGC and TGLCC. For instance, if savings in TGC is highly important, then choosing seven lines for dynamic rating seems reasonable. Nevertheless, if minimum number of lines for dynamic rating is highly desirable, then choosing one or two lines for dynamic rating seems to be a good decision.

## 5. Conclusions

In this paper we present a three-layer min-max-min robust optimization framework and find critical (and minimum number of) lines for dynamic rating operations. The goal is to increase the security of the power system against the worst-case N-K contingency case as much as possible. To find the minimum number of lines for DLR we decompose the problem into a master problem and a slave problem. The master problem finds critical lines for dynamic rating for the economic power dispatch. The slave problem evaluates the performance of selected dynamic rating lines in terms of the generation and load curtailments under N-K contingency case. Our numerical experiments on two test power systems shows the effectiveness of our proposed model and solution approach. For instance, in 118-bus test system, we reach almost 24% reduction in

generation and load curtailments by applying dynamic line rating only on two lines. In addition, the computational time required to find optimal solution is short and practical for the industry applications. Future work consists of incorporating stochastic nature of wind farms and dynamic line ratings into the proposed model. Also, optimal transmission switching (Fisher et al. 2008) can be considered in addition to dynamic line rating to reach better solutions.

## Conflict Of Interest

The authors declare no conflict of interest.

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